2025.M30

2025L003A2EL

Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate Examination 2025

Mathematics

Paper 2

Higher Level

Monday 9 June Morning 9:30 - 12:00

300 marks

Examination Number

Date of Birth

For example, 3rd February 2005 is entered as 03 02 05

Centre Stamp

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Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	4 questions

Answer **any five** questions from Section A.
Answer **any three** questions from Section B.

Write your Examination Number in the box on the front cover.

Write your answers in blue or black pen. You may use pencil in graphs and diagrams only.

This examination booklet will be scanned and your work will be presented to an examiner on screen. Anything that you write outside of the answer areas may not be seen by the examiner.

Write all answers into this booklet. There is space for extra work at the back of the booklet.
If you need to use it, label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if your solutions do not include relevant supporting work.

You may lose marks if the appropriate units of measurement are not included, where relevant.

You may lose marks if your answers are not given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 marks

Answer **any five** questions from this section.

Question 1 **(30 marks)**

(a) $p \in \mathbb{R}$ is a constant.

The point $(p, 5)$ lies on the line $3x - 2y + 28 = 0$.
Find the value of p.

$p =$ ____________

(b) The line l has equation $y = -\frac{1}{3}x + 11$.

The line h has equation $2x - 5y + 10 = 0$.

Work out the size of the acute angle between the lines l and h.
Give your answer correct to the nearest degree.

(c) A line cuts the x-axis at the point $A(a, 0)$ and the y-axis at $B(0, b)$, where $a, b \in \mathbb{Z}$.

The slope of this line is $-\frac{2}{3}$.
The area of the triangle enclosed by this line, the x-axis, and the y-axis is 12 square units.

There are **two** different lines that satisfy these conditions.
Find the equation of each of these lines.

It may be useful to draw a diagram.

Line 1:

Line 2:

Question 2 **(30 marks)**

(a) A circle s has the equation $(x-4)^2+(y+2)^2=45$.

(i) Write down the centre and radius of the circle s.

Centre = (,) Radius = ____________

(ii) Find the equation of the tangent to s at the point $(-2,-5)$.
Write your answer in the form $y=mx+c$, where $m, c \in \mathbb{Z}$.

(b) The circle t has the following equation, where $k \in \mathbb{R}$ is a constant:

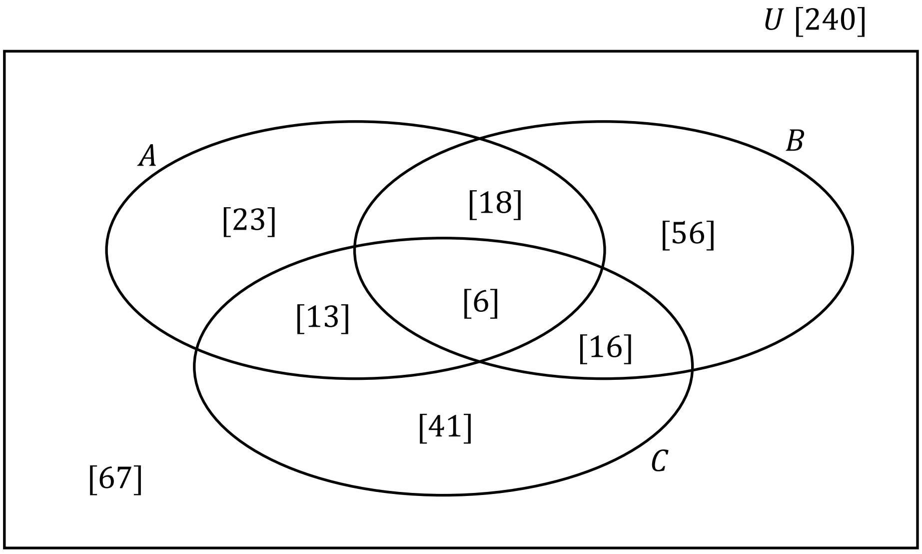
$$x^2 + y^2 + 28x - 46y + k = 0$$

The horizontal line $y = k$ is a tangent to the circle t.
Find the two possible values of k.

$k =$ ____________ or ____________

Question 3 **(30 marks)**

(a) 240 people were surveyed about which of three countries, A, B, or C, they had been to. The Venn diagram below shows the number of people who had been to each combination of these countries, as well as those who had been to none of the three.



In this question, the event A is the event that a person picked at random from the 240 people surveyed had been to country A, and so on.

(i) Show that $P(A) = \frac{1}{4}$.

(ii) Verify that, for the values in this diagram:

$$P(A \cup C) = P(A) + P(C) - P(A \cap C)$$

(iii) Are A and B independent events? Use calculations to justify your answer.

Answer:

Justification:

(b) Two of the 240 people are picked at random.

Find the probability that one of them had been to all three countries, and the other had been to none of the three countries. Give your answer as a fraction in its simplest form.

Question 4 **(30 marks)**

(a) The ages of twelve people in a class are given below.
They are in ascending order, and $x \in \mathbb{N}$.

11	12	12	14	15	x	18	18	19	22	25	30

(i) The median age is $17{\cdot}5$.
Find the value of x.

(ii) The first quartile (Q_1) is 13.
Work out the interquartile range of the ages.

(b) Michael finds the mean and the median of a list of 10 numbers.
He then changes the biggest number in the list to make it even bigger.

Will this change the mean, the median, or both?
Justify your answer fully.

This will change:	the mean only	the median only	both the mean and the median
Tick (✓) **one** box only	☐	☐	☐

Justification:

(c) The frequency table below shows the ages of the people in another class, where $k \in \mathbb{N}$.

Age (years)	$24-30$	$30-36$	$36-42$	$42-48$	$48-54$	$54-60$
Number of people	4	5	9	k	4	2

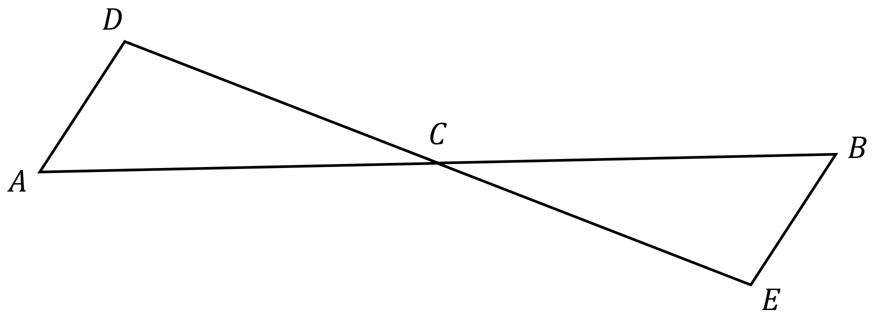
Note: $24-30$ means "at least 24, and less than 30", and so on.

Using mid-interval values, the mean age of the people in the class is $40{\cdot}4$ years, based on the data in the table above.

Use this to work out the value of k.

Question 5 **(30 marks)**

(a) The diagram shows two triangles, ACD and BCE.
C is the midpoint of $[AB]$.
AD is parallel to EB, and the point C lies on DE.

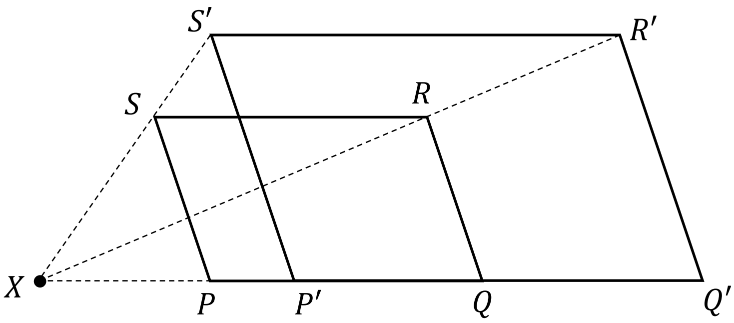


Prove that the triangles ACD and BCE are congruent.
Give a reason for each statement that you make in your proof.

(b) The parallelogram $PQRS$ is shown on the right.

The point X lies on the line PQ, as shown.

$P'Q'R'S'$ is an enlargement of $PQRS$, using point X as the centre of enlargement.



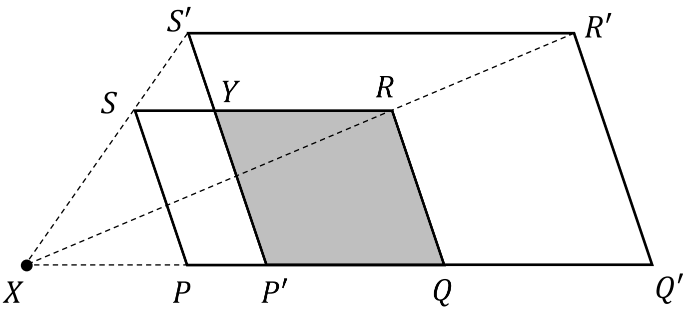
(i) $|XQ| = 8$ cm and $|QQ'| = 4$ cm.

Show that the scale factor of the enlargement, k, is $1{\cdot}5$.

(ii) $PQRS$ and $P'Q'R'S'$ are shown again in the diagram on the right.

Y is the point where the lines SR and $P'S'$ intersect. The region $P'QRY$ is shaded.

$|XP| = 3$ cm.
The area of $PQRS$ is 20 cm^2.



Use this to find the area of the shaded region $P'QRY$, in cm^2.

Question 6 **(30 marks)**

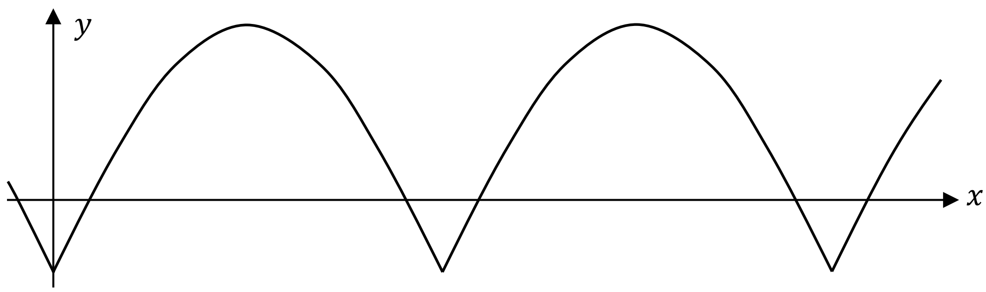
(a) Find **all six** solutions to the following equation in A, where $-360° \leq A \leq 720°$:

$$\sin A = \frac{1}{2}$$

(b) $f(x)$ is the following function, where $x \in \mathbb{R}$ is in radians:

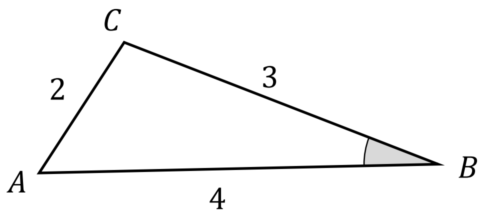
$$f(x) = |4\sin x| - 1$$

Part of the graph of $y = f(x)$ is shown below.
Write down the period and range of $f(x)$.



Period = ____________ Range = [,]

(c) In the triangle ABC, $|AC| = 2$, $|BC| = 3$, and $|AB| = 4$.



Use the **Cosine Rule** to find the value of $\tan \angle CBA$, without using a calculator.

Give your answer in the form $\frac{\sqrt{n}}{m}$, where $n, m \in \mathbb{Z}$. Show all your working out.

$\tan \angle CBA =$ ____________

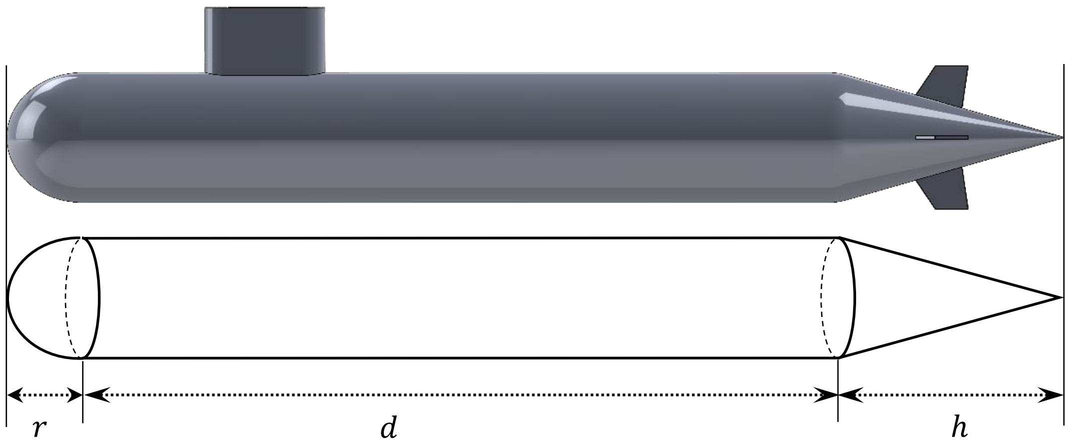
Section B	**Contexts and Applications**	**150 marks**

Answer **any three** questions from this section.

Question 7 **(50 marks)**

(a) Below is a scaled diagram of a submarine.

The body of the submarine is roughly in the shape of a cylinder, with a cone at one end and a hemisphere at the other end, as shown.



(i) **Measure** the lengths labelled r, d, and h on the diagram above.
Write the length of each, correct to the nearest cm, in the table below.

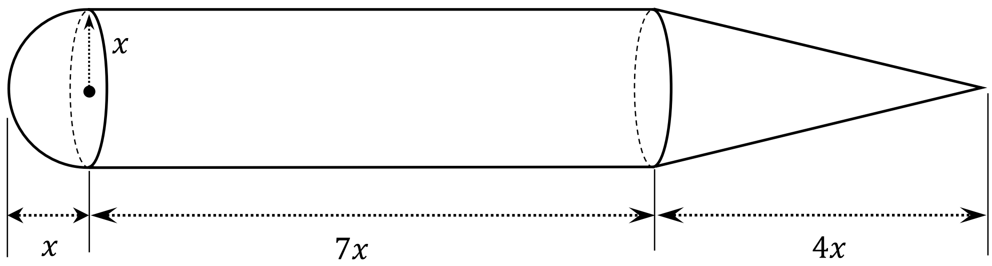
Label	r	d	h
Length on diagram (cm)			

The **actual** total length of the submarine shown in the scaled diagram above is **90 m**.

(ii) Use the measurements from part **(a)(i)** to work out the **actual** lengths represented by r, d, and h. Give each value in metres, correct to 1 decimal place.

Label	r	d	h
Actual length (metres)			

(b) The diagram below shows the body of a **different** submarine with a similar design.
The dimensions of this submarine are all given in terms of x, where $x \in \mathbb{R}$.
The hemisphere, cylinder, and cone all have a radius of x.



This submarine has a **volume** of 6738 m^3, correct to the nearest m^3.

By solving an equation in x, find the **total length** of this submarine.
Give your answer in metres, correct to 1 decimal place.

Total length = ________________

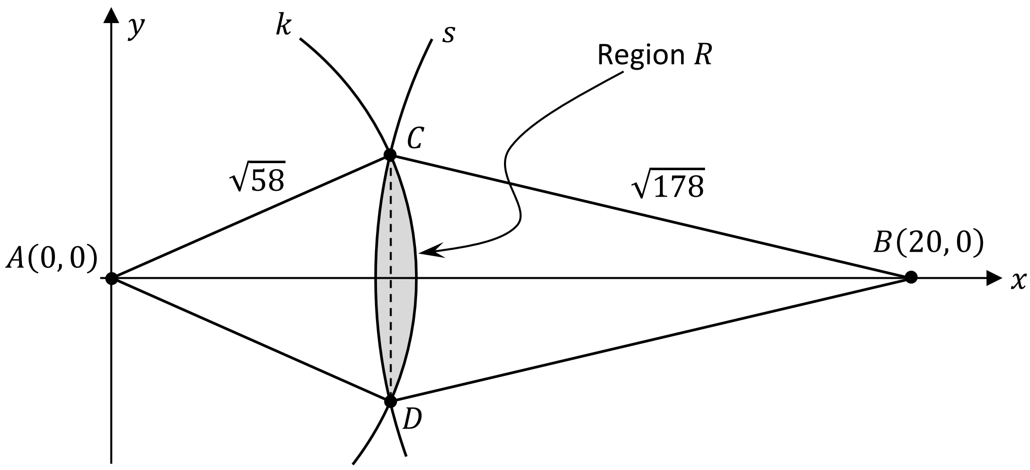
This question continues on the next page.

(c) A submarine is within $\sqrt{58}$ km of a point A and is within $\sqrt{178}$ km of a point B.
The distance from A to B is 20 km. A, B, and the submarine are all at the same depth.

This is represented on the co-ordinate diagram below.
The points $A(0, 0)$ and $B(20, 0)$ are shown.

Part of the circles k and s are shown, where k has centre $A(0, 0)$ and radius $\sqrt{58}$, and s has centre $B(20, 0)$ and radius $\sqrt{178}$. The circles intersect at the points C and D.

The submarine is in the shaded region, R, the region that is inside **both** circles k and s.



(i) Write down the equation of the circle s.

The point C is $(7, 3)$ and the point D is $(7, -3)$.

(ii) By using your answer to part **(a)(i)**, or otherwise, verify that $(7, 3)$ lies on the circle s.

(iii) By using $C(7, 3)$ and $D(7, -3)$, find the area of the triangle DBC, in km^2.

(iv) Find the size of the acute angle $\angle CBD$, correct to the nearest degree.

(v) The area of the **sector** ADC of the circle k is $23{\cdot}4837$ km^2, correct to 4 decimal places.
The area of the **triangle** ADC is 21 km^2.

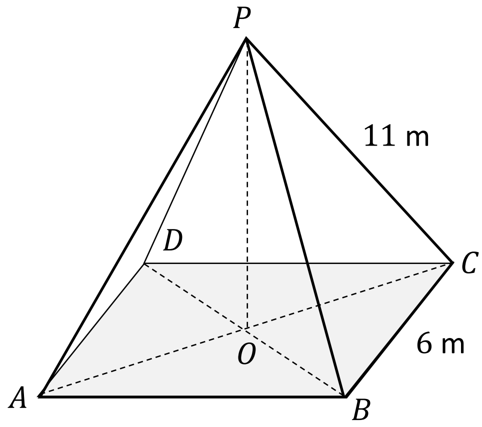
Using this, work out the area of the shaded region, R, that is inside **both** circles.
Give your answer in km^2, correct to 2 decimal places.

Question 8 **(50 marks)**

(a) A roof is in the shape of a square-based pyramid, as shown. The square base, $ABCD$, has sides of length 6 m.

The diagonals of $ABCD$ meet at the point O.
The top of the pyramid, P, is directly above O.

The four triangular faces are congruent to each other, with $|AP| = |BP| = 11$ m.



(i) Use the theorem of Pythagoras to show that $|OB| = 3\sqrt{2}$ m, **and** hence find the value of $|OP|$, the vertical height of the pyramid.

Give $|OP|$ in surd form.

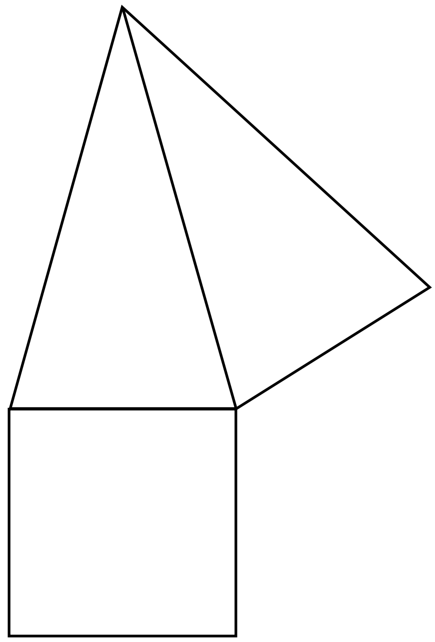
Show $\lvert OB\rvert = 3\sqrt{2}$ m:	Find $\lvert OP\rvert$:

(ii) On the triangular face PAB, the size of $\angle PAB$ is $74{\cdot}2°$, correct to 1 decimal place.

Using this, or otherwise, work out the **total area** of the four triangular faces of the roof. Give your answer correct to the nearest m^2.

(iii) The diagram below shows part of a scaled diagram of the **net** of this pyramid.
The diagram shows the square base and two of the triangular sides.

Construct the rest of the scaled diagram of the net of the pyramid.
Show all construction lines clearly.



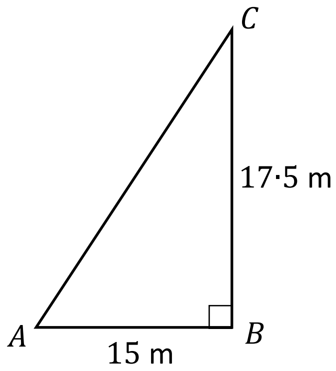
This question continues on the next page.

(b) In the diagram on the right, $[BC]$ represents a flagpole.

$|AB| = 15$ m and $|BC| = 17{\cdot}5$ m, as shown.
AB is perpendicular to BC.

Ally measures the size of $\angle CAB$, the angle of elevation of the flagpole. She makes a mistake, and measures that $|\angle CAB|$ is $\mathbf{52}°$, which is **not** correct.

Work out the **percentage error** in Ally's value for $|\angle CAB|$.
Give your answer correct to 1 decimal place.

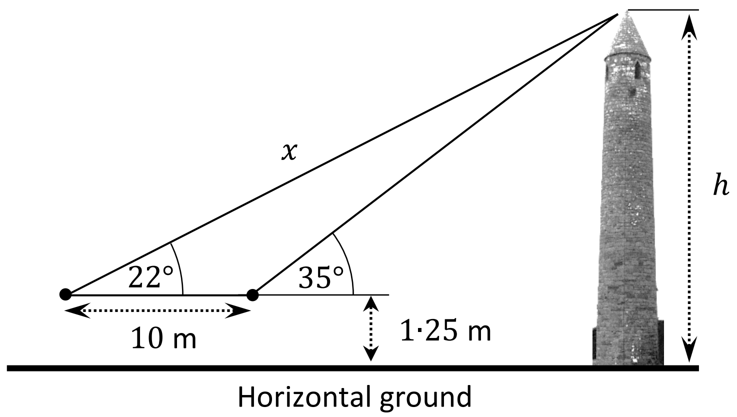


(c) Ally is also working out the height of a round tower.

She measures the angle of elevation to the top of the tower.
She then moves 10 m away from the tower, on horizontal ground, and measures the angle of elevation of the top of the tower again.

She measures both of these angles of elevation from a height of $1{\cdot}25$ m.

She draws the diagram below to show her measurements.
Two lengths, x and h, are shown in the diagram.



(i) Use the **Sine rule** to show that $x = 25{\cdot}5$ m, correct to 1 decimal place.

(ii) Hence, find the **total** height of the tower, marked h in the diagram on the previous page. Give your answer in metres, correct to 1 decimal place.

Question 9 **(50 marks)**

In parts **(a)** and **(b)** of this question, give answers **correct to 4 decimal places**, where relevant.

(a) Assume that $6{\cdot}7\%$ of people in Ireland have diabetes.

For a particular test for diabetes, each person tests either positive or negative.
The probability that someone who **has** diabetes gets a correct positive result is 99%.

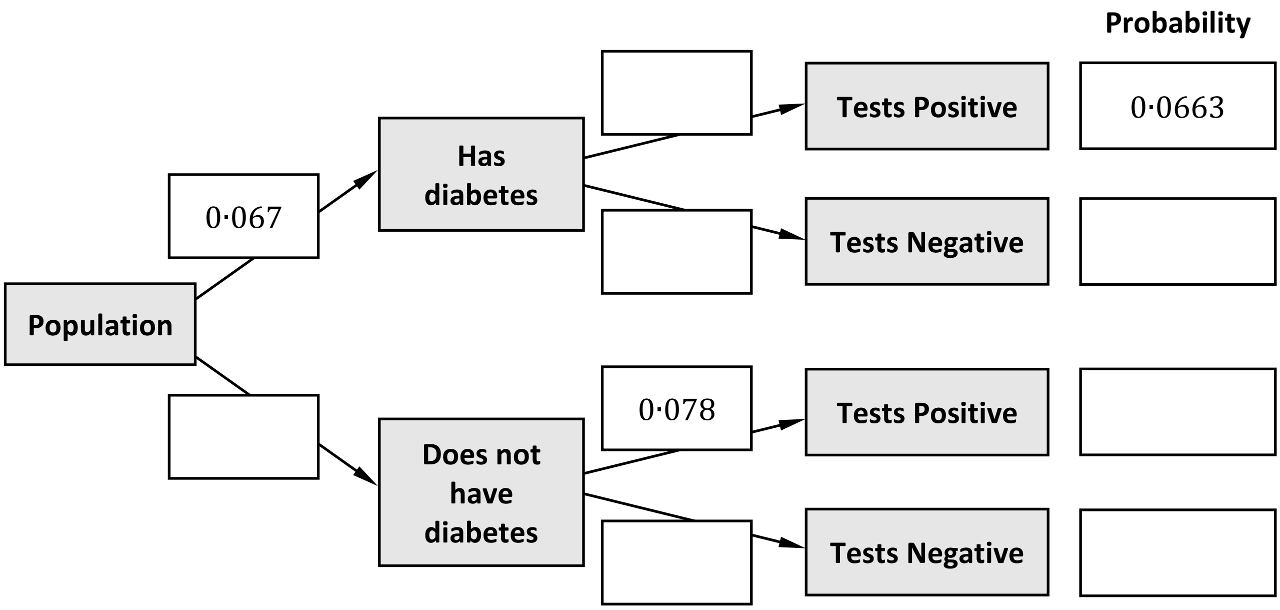
However, the probability that someone who does **not** have diabetes gets an incorrect positive result is $7{\cdot}8\%$.

One person is picked at random from the people in Ireland, and is tested for diabetes using this test. Use the information above to complete the tree diagram below, by:

(i) writing the proportion associated with each branch of the tree diagram into the appropriate box **and**

(ii) working out the probability for each outcome and writing it in the appropriate box in the final column.

Some values are already filled in.
They have been given correct to 4 decimal places, where relevant.



Use the probabilities from the tree diagram to answer parts **(a)(iii)** and **(a)(iv)**.

(iii) Find the probability that the person picked at random tests positive for diabetes.

(iv) The person picked at random tests positive for diabetes, using this test.
Find the probability that they actually have diabetes.

(b) 5 people are picked at random from the people in Ireland.

Assuming that $6 \cdot 7\%$ of people in Ireland have diabetes, work out the probability that **2 or more** of these 5 people have diabetes.

This question continues on the next page.

(c) 20 people take part in a clinical trial.
10 of them will be picked to be in group **A**. The remaining 10 people will be in group **B**.

(i) How many different combinations of 10 people can be picked to be in group **A**?

10 people are picked and are put in group **A**.

(ii) Each person in group **A** is now paired with a person in group **B**, making 10 pairs.
How many different sets of 10 pairs can be made?

(d) In a different clinical trial, 24 people are split into two groups, **X** and **Y**.
8 people are in group **X** and 16 people are in group **Y**.

Each person in group **X** is paired with two people in group **Y**,
so that everyone in group **Y** is in exactly one of these pairings.

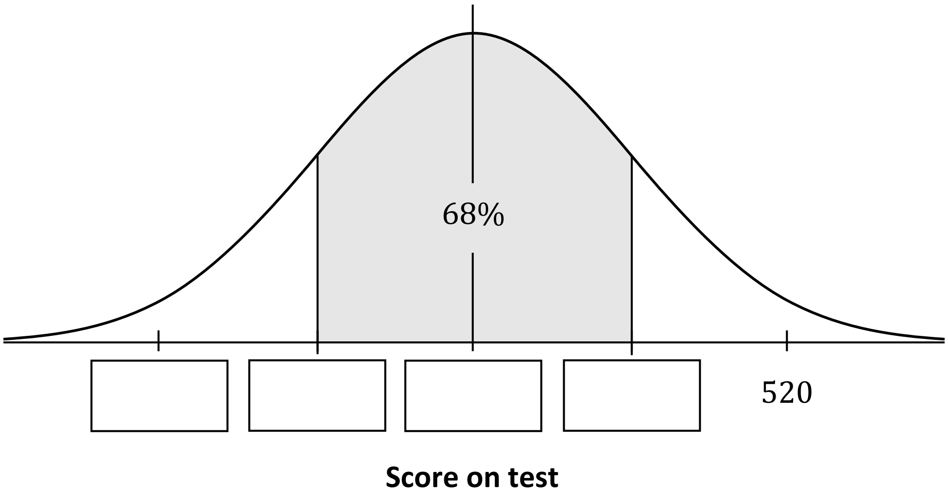
How many different sets of such pairings can be made? Give your answer in the form $a \times 10^n$ where $1 \leq a < 10$, $n \in \mathbb{N}$, and a is correct to 3 decimal places.

Question 10 **(50 marks)**

A particular test is used to measure how well students around the world can do maths problems.

(a) Worldwide, scores on this test are normally distributed with a mean score of 400 and a standard deviation of 60.

(i) The scaled diagram below shows the distribution of scores on this test.
Use the **empirical rule** to fill in the four missing values in the diagram.



(ii) Use the normal distribution to work out the proportion of students worldwide who would score above 420 on the test. Give your answer correct to 2 decimal places.

This question continues on the next page

A random sample of students is taken from each country that takes part in the maths test.

The table below shows the mean score and the standard deviation for this sample from three of these countries, labelled **X**, **Y**, and **Z**. It also shows the size of each of these samples.

Country	Mean score of sample	Standard deviation of sample	Size of sample
X	387	66·2	2161
Y	403	70·6	2724
Z	396	53·7	2520

For parts **(b)** and **(c)**, use the relevant standard deviation from the table above.

(b) Using values from the table, construct a 95% confidence interval for the population mean score of country **X**. Give each value correct to 1 decimal place.

(c) For country **Y**, researchers carried out a hypothesis test at the 5% level of significance to see if the population mean score of the country was different to 400 (the worldwide mean).

The null hypothesis was that the mean score for country **Y** was 400.
The alternative hypothesis was that it was **not** 400.

(i) Using values from the table, work out the test statistic (z-score) of the sample mean for country **Y** for this test. Give your answer correct to 2 decimal places.

(ii) Hence, work out the p-value of this test statistic **and** state the conclusion of the hypothesis test in the given context, making reference to the mean score for country **Y**.

p-value:

Conclusion:

(d) In country **Z**, 50% of all students have a pet in their home and 50% do not.

Describe how the sample of 2520 students from country **Z** could be taken as a **stratified random sample** with respect to having a pet in the home, **and** explain why it is probably **not** useful to do this when looking at these students' maths scores.

How to take as a stratified random sample:

Why this would **not** be useful:

This question continues on the next page.

(e) For one of the questions on the test, students are given a mark of 0, 1, 2, or 3.

The proportion of students who get each mark is shown in the table below.
Here, $p, r \in \mathbb{R}$ and $p, r \geq 0$.

Mark	0	1	2	3
Proportion	$0{\cdot}19$	p	$2r$	r

A student is picked at random.
The expected value of their mark for this question will depend on the values of p and r.
Find the **largest** value that the expected value of their mark could be.

You may use this page for extra work.
Label any extra work clearly with the question number and part.

Acknowledgements

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Leaving Certificate – Higher Level

Mathematics Paper 2

Monday 9 June

Morning 9:30 - 12:00