2020 Question 1 – Higher Paper 1 Question

Question 1 **(25 marks)**

(a) $f(x) = x^2 + 5x + p$ where $x \in \mathbb{R}$, $-3 \leq p \leq 8$, and $p \in \mathbb{Z}$.

(i) Find the value of p for which $x + 3$ is a factor of $f(x)$.

(ii) Find the value of p for which $f(x)$ has roots which differ by 3.

(iii) Find the two values of p for which the graph of $f(x)$ will not cross the x-axis.

(b) Find the range of values of x for which $|2x + 5| - 1 \leq 0$, where $x \in \mathbb{R}$.

2020 Question 1 – Higher Paper 1 Solution

(a) (i) If $x+3$ is a factor, $x=-3$ is a root $\Rightarrow$ $f(-3)=0$

$f(-3)=(-3)^2+5(-3)+p=0 \;\Rightarrow\; 9-15+p \;\Rightarrow\; p=6$

$\therefore\; f(x)=x^2+5x+6=0$ **(5 marks)**

(ii) Call the roots h and $h+3$

We know that the sum of the roots of $ax^2+bx+c=0$ is $-\dfrac{b}{a}$

and the product of the roots is $\dfrac{c}{a}$

$\therefore h+h+3=-5 \;\Rightarrow\; 2h=-8 \;\Rightarrow\; h=-4,\;\; h+3=-1$

$p=(-4)(-1)=4$ **(10 marks)**

(iii) If the graph of $f(x)$ does not cross the x axis

that implies $f(x)=0$ has no real roots

$ax^2+bx+c=0$ has no real roots if $b^2-4ac<0$

$a=1,\;\; b=5,\;\; c=p \;\Rightarrow\; 25-4p<0$

Solve $25-4p<0 \;\Rightarrow\; 25<4p,\;\; 6.25<p,\;\; p=7,\;\; p=8$

as p is an integer and $p\le 8$ **(5 marks)**

(b) Solve $|2x+5|-1\le 0 \;\Rightarrow\; |2x+5|\le 1$

$\Rightarrow\; (2x+5)^2\le 1 \;\Rightarrow\; 4x^2+20x+25-1\le 0$ (quadratic inequality)

Solve $4x^2+20x+24=0 \;\Rightarrow\; x^2+5x+6\le 0 \;\Rightarrow\; (x+3)(x+2)=0$

$\Rightarrow\; x=-3,\;\; x=-2$ Plot points (include zero)

−3 −2 0

Check zero in the inequality

$|2x+5|\le 1,\;\; |2(0)+5|\le 1$ false $\therefore$ zero is not part of the solution

$\therefore\; -3\le x\le -2$ **(5 marks)**

Comment: *Very long and tested all knowledge of quadratic equations.*

2020 Question 2 – Higher Paper 1 Question

Question 2 **(25 marks)**

(a) Find the two complex numbers z_1 and z_2 that satisfy the following simultaneous equations, where $i^2 = -1$:

$$iz_1 \quad = -4 + 3i$$
$$3z_1 - z_2 = 11 + 17i.$$

Write your answers in the form $a + bi$ where $a, b \in \mathbb{Z}$.

(b) The complex numbers $3 + 2i$ and $5 - i$ are the first two terms of a **geometric** sequence.

(i) Find r, the common ratio of the sequence.
Write your answer in the form $a + bi$ where $a, b \in \mathbb{Z}$.

(ii) Use de Moivre's Theorem to find T_9, the ninth term of the sequence.
Write your answer in the form $a + bi$, where $a, b \in \mathbb{Z}$.

2020 Question 2 – Higher Paper 1 Solution

(a) Given $iz_1 = -4+3i$, $3z_i - 2z_2 = -11+17i$ and $i^2 = -1$

If $iz_1 = -4+3i \Rightarrow i^2 z_1 = -4i + 3i^2 \Rightarrow -z_1 = -4i - 3$ (multiply everything by i)

$\Rightarrow z_1 = 4i + 3$, $z_1 = 3 + 4i$

and $3z_1 - z_2 = 11 + 7i \Rightarrow 3(4i+3) - z_2 = 11 + 17i$

$\Rightarrow 12i + 9 - z_2 = 11 + 17i$

$12i + 9 - 11 - 17i = z_2 \Rightarrow z_2 = -2 - 5i$ **(10 marks)**

(b) If $3+2i$ and $5-i$ are in geometric sequence, then the common ratio is $\frac{t_2}{t_1} = r$

(i) $\frac{5-i}{3+2i} = \frac{5-i}{3+2i} \cdot \frac{3-2i}{3-2i} = \frac{15-10i-3i-2}{13} = \frac{13-13i}{13} = 1-i$ (r)

The 9[th] term of a geometric series is ar^8 ($t_n = ar^{n-1}$)

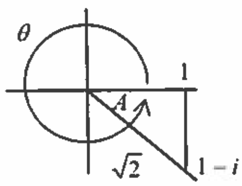
$a = 3+2i$, $r = (1-i)$ **(5 marks)**

(ii) We want to find t_9 using De Moivre's theorem

Use the POL(+) key on the Casio. It is above the + key

Input as follows:

1. Shift+ to get POL(
2. Input 1,−1 (the comma is shift right bracket)
3. = to get $1.4142(\sqrt{2})$ and $-45° = 315$
4. This gives $\sqrt{2}(\cos 315 + i \sin 315)$



$1 - i = \sqrt{2}(\cos 315 + i \sin 315)$

Or $\tan A = \frac{1}{1} \Rightarrow A = 45° \Rightarrow \theta = 315$

$\therefore 1 - i = \sqrt{2}(\cos 315 + i \sin 315)$

$(1-i)^8 = (\sqrt{2})^8 (\cos 315 + i \sin 315)^8 = 16(\cos 2520 + i \sin 2520) = 16(1) = 16$

$\therefore t_9 = (3+2i)(16) = 48 + 32i$ **(10 marks)**

Comment: *Nice question, especially part (b).*